# CONFINEMENT AND FLAVOR SYMMETRY BREAKING VIA MONOPOLE CONDENSATION

HITOSHI MURAYAMA

*Department of Physics, University of California, Berkeley CA 94720, USA*

*Lawrence Berkeley National Laboratory, Cyclotron Road, Berkeley CA94720 USA*
*E-mail: murayama@lbl.gov*

We discuss dynamics of $N = 2$ supersymmetric $SU(n_c)$ gauge theories with $n_f$ quark hypermultiplets. Upon $N = 1$ perturbation of introducing a finite mass for the adjoint chiral multiplet, we show that the flavor $U(n_f)$ symmetry is dynamically broken to $U(r) \times U(n_f - r)$, where $r \leq [n_f/2]$ is an integer. This flavor symmetry breaking occurs due to the condensates of magnetic degrees of freedom which acquire flavor quantum numbers due to the quark zero modes. We briefly comment on the $USp(2n_c)$ gauge theories. This talk is based on works with Giuseppe Carlino and Ken Konishi.[1,2]

## 1 Introduction

There have been at least two main dynamical issues in gauge theories: confinement and flavor symmetry breaking. The former is an obvious requirement in understanding the real-world strong interaction dynamics, namely the lack of observation of isolated quarks. The spectrum of light hadrons demands a linear potential with respect to the distance beween the quark and the antiquark in meson boundstates. The latter is a more subtle issue. Nambu pointed out that the lightness of the pions can be understood if they are what we now call Nambu–Godstone bosons of spontaneously broken symmetries. We need the $SU(3)_L \times SU(3)_R$ flavor symmetry of the QCD to be dynamically broken down to $SU(3)_V$ by quark bilinear condensates

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle \neq 0. \tag{1}$$

An important question is what microscopic mechanism is behind the confinement and dynamical flavor symmetry breaking. The seminal work by Seiberg and Witten[3] showed that, using $N = 2$ supersymmetric gauge theories, confinement can be understood as a consequence of the magnetic monopole condensation as conjectured a long time ago by ‘t Hooft and Mandelstam.[4] Our aim is to bring the understanding of the dynamical flavor symmetry breaking to the same level, done in collaboration with Giuseppe Carlino and Ken Konishi[1,2]. Surprisingly, this question had not been addressed systematically so far. Seiberg and Witten themselves studied the case with flavor,[5] but there were only two examples which exhibited dynamical flavor symmetry breaking ($SU(2)$ with $n_f = 2, 3$) and it was not possible to draw a general lesson. Later works on general gauge groups[6,7] focused on the appearance of the dual gauge group, and did not discuss the issue of flavor symmetry breaking.

We start with $N = 2$ supersymmetric $SU(n_c)$ QCD with $n_f$ hypermultiplet quarks in the fundamental representation. We later add a perturbation which leaves only $N = 1$ supersymmetry, $W = \mu \mathrm{tr}\Phi^2$, a mass term for the adjoint chiral superfield in the $N = 2$ vector multiplet. This theory has $U(n_f)$ flavor symmetry. We found that the flavor symmetry is in general dynamically broken as

$$U(n_f) \to U(r) \times U(n_f - r). \tag{2}$$

There are isolated vacua for $0 \leq r \leq [n_f/2]$. We have shown that this dynamical flavor symmetry breaking is caused by condensation of magnetic degrees of freedom. For the vacuum $r = 0$, there is no breaking of the flavor $U(n_f)$ symmetry. For the vacuum $r = 1$, what condenses is nothing but the magnetic monopoles, which belong to the fundamental representatin of the $U(n_f)$ flavor group. For the vacua $r > 1$, magnetic monopoles “break up” into “dual quarks” before reaching the singularities where they become massless; it is the "dual quark" which condenses and breaks the flavor symmetry. In any case, the flavor symmetry breaking and the confinement[a] are both caused by the condensation of magnetic degerees of freedom.

Thanks to holomorphy, there is no phase transition by varying $\mu$ from small ($\mu \ll \Lambda$) to large ($\mu \gg \Lambda$). Therefore one can study the theory in both limits and compare the results; this would not only provide us non-trivial cross checks but also insight into the dynamics of the theory. We can also resort to completely different techniques to analyze the theory in the different limits.

In the limit (1) ($\mu \gg \Lambda$), we can integrate the adjoint chiral multiplet $\Phi$ out from the theory, and study the resulting $N = 1$ low-energy theory. The low-energy theory has a superpotential term

$$W = -\frac{1}{\mu}(\tilde{Q}T^a Q)(\tilde{Q}T^a Q), \qquad (3)$$

where $Q$ ($\tilde{Q}$) are the quark chiral superfields in the fundamental (anti-fundamental) representation of the gauge group. Then we can use analysis by Seiberg[9] on $N = 1$ supersymmetric QCD together with the above effective superpotential (3). It is then easy to identify the vacua of the theory by solving for the extrema of the superpotential with respect to the gauge-invariant composites such as $M^{ij} = \tilde{Q}^i Q^j$ or $B^{i_1\cdots i_{n_c}} = Q^{i_1}Q^{i_2}\cdots Q^{i_{n_c}}$, $\tilde{B}^{i_1\cdots i_{n_c}} = \tilde{Q}^{i_1}\tilde{Q}^{i_2}\cdots\tilde{Q}^{i_{n_c}}$. This makes it easy to identify the flavor symmetry breaking patterns.

In the other limit (2) ($\mu \ll \Lambda$), we start with $N = 2$ limit ($\mu = 0$) where the low-energy effective theory is known exactly. In this limit, we can identify monopole degrees of freedom etc which become massless at singularities. We then turn on $\mu \neq 0$. This way we obtain information on the microscopic dynamics of magnetic degrees of freedom.

When considering the theory in various limits, a very powerful check is provided by counting the number of vacua, by further perturbing the theory by finite masses of hypermultiplet quarks. Quark masses make the vacua discrete and countable, and we must obtain the same number of vacua in different limits. In fact, we considered four such limits in total. Two of them have large $\mu$. In the limit (1A), we regard both $\mu$ and $m_i$ large and solve for vacua semi-classically (*i.e.*, including the effects of gaugino condensates in unbroken pure Yang-Mills factors). In the limit (1B), we integrate out $\Phi$ and use known $N = 1$ dynamics together with the effective superpotential Eq. (3), further combined with the mass terms for the quarks. The other two have small $\mu$, namely setting $\mu = 0$ first, and then reintroduce $\mu \neq 0$ later on. In the limit (2A), we approach singularities from large $\Phi$ on the Coulomb branch. In the limit (2B), we approach singularities from large $Q$, $\tilde{Q}$ on the Higgs branch. All these approaches should give the identical number of vacua, and the consistency among them tell us, for example, which singularity on the Coulomb branch corresponds to which symmetry breaking pattern.

I will not discuss the limit (1A) in this talk and simply refer interested parties to our paper.[2] I first discuss the limit (1B) and identify the flavor symmetry breaking patterns. Then I will briefly review how the monopoles acquire flavor quantum numbers. I will move on to the analyses with small $\mu$ next, first on the Coulomb branch, and next on the Higgs branch. The consistency among different approaches gives us full understanding of the dynamics.

## 2 Large $\mu$ Analysis

$N = 2$ supersymmetric QCD can be viewed as a special version of $N = 1$ supersymmetric gauge theories with the following superpotential

$$W = \sqrt{2}\tilde{Q}_i \Phi Q_i + m_i \tilde{Q}_i Q_i + \mu \mathrm{tr}\Phi^2, \qquad (4)$$

where the last term breaks $N = 2$ to $N = 1$. When $\mu$ is large, we can integrate out $\Phi$ field, and obtain

$$W = -\frac{1}{\mu}(\tilde{Q}_i T^a Q_i)(\tilde{Q}_j T^a Q_j) + m_i \tilde{Q}_i Q_i. \quad (5)$$

[a] We use the term "confinement" somewhat loosely, as in "s-confinement" in [8].

Doing Fierz transformation on the first term, we obtain

$$W = \frac{1}{2\mu}\left[\mathrm{tr}M^2 - \frac{1}{n_c}(\mathrm{tr}M)^2\right] + \mathrm{tr}mM, \quad (6)$$

where $M_{ij} = \tilde{Q}_i Q_j$ is the meson chiral superfield, and the mass $m = \mathrm{diag}(m_1, \cdots, m_{n_f})$ and the meson field are in the matrix notation.

Due to lack of time, I concentrate on the case $n_f < n_c$. I refer to our papers[1,2] for larger number of flavors. In this case, the non-perturbative superpotential[10] is added to Eq. (6):

$$\Delta W = (n_c - n_f)\frac{\Lambda_1^{(3n_c-n_f)/(n_c-n_f)}}{(\mathrm{det}M)^{1/(n_c-n_f)}}, \quad (7)$$

where $\Lambda_1^{3n_c-n_f} = \mu^{n_c}\Lambda^{2n_c-n_f}$ is the scale of the low-energy $N=1$ theory.

By solving for the meson matrix $M = \mathrm{diag}(\lambda_1, \cdots, \lambda_{n_f})$, we find that $\lambda_i$ satisfy quadratic equations and hence there are two solutions for each of them. We obtain

$$\lambda_i = \frac{1}{2}(Y \pm \sqrt{Y^2 + 4\mu X}) + O(m), \quad (8)$$

where the signs $\pm$ indicate two solutions for each $i = 1, \cdots, n_f$ and hence there are $2^{n_f}$ possibilities. For the choice of $r$ plus signs and $n_f - r$ minus signs, we can further determine $X$ and $Y$, which can take $(2n_c - n_f)$ possible phases. Avoiding double counting for $r \leftrightarrow n_f - r$, we find $(2n_c - n_f)2^{n_f-1}$ vacua in total. The most important outcome from this analysis is that, in $m \to 0$ limit, $r$ eigenvalues with plus sign are degenerate, and $n_f - r$ eigenvalues with minus sign are also degenerate. Such a vacuum for the meson field exhibits dynamical flavor symmetry breaking, $U(n_f) \to U(r) \times U(n_f - r)$.

## 3 Semi-classical Monopoles

As was shown by 't Hooft and Polyakov, there are solitonic solutions to the gauge-Higgs system which appear as magnetic monopoles under the low-energy gauge group. The canonical example is the $SU(2)$ gauge theory with the adjoint Higgs $\Phi$, where the expectation value of $\Phi = a\sigma_3$ breaks $SU(2) \to U(1)$. The mass of the magnetic monopole is given roughly as $M \sim 4\pi a/g$, while the mass of the $W$-boson is $m_W \sim ga$. Therefore for the weakly-coupled case, the magnetic monopole is heavy and $W$ (electric monopole) is light, while for the strongly-coupled case, the magnetic monopole is light and the $W$-boson is heavy.

The case with flavor is quite interesting.[11] The cancellation of the $SU(2)$ Witten anomaly requires an even number of flavors: $2n_f$ doublet quarks. They can couple to the adjoint Higgs as $\mathcal{L}_{\mathrm{Yukawa}} = q_i \Phi q_i$, which produces Majorana-type mass terms. The largest possible flavor symmetry is $SO(2n_f)$. Solving Dirac equation for the quarks in the monopole background, there is one zero-energy mode for each flavor. The important question is what statistics the fermion zero modes follow. Surprisingly, they are bosons. The reasoning is simple. The way to judge if an excitation is bosonic or fermionic is by studying the $2\pi$ rotation of space and asking if the excitation produces a minus sign (fermion) or not (boson). In the presence of a 't Hooft–Polyakov monopole, a naive $2\pi$ spatial rotation is not a symmetry because the isospin space is tied to the real space ("hedgehog"). Therefore one needs to make a $2\pi$ rotation both for the real space and the isospin (gauge) space to determine statistics. For fermion zero modes in the $SU(2)$ doublet representation, spatial rotation produces a minus sign, while the isospin rotation produces another minus sign. The fermion zero mode does not produce a minus sign under the true $2\pi$ rotation and hence is a boson. Therefore monopole states with or without the fermion zero mode have the same statistics and the same energy. In other words, the monopole states form a multiplet.

For the $SU(2)$ gauge theory, or in general $USp(2n_c)$ gauge theories, we have $SO(2n_f)$ flavor symmetry. Fermion zero mode operators $q^i$ follow the anti-commutation relation $\{q^i, q^j\} = \delta^{ij}$ upon canonical quantization, and they are represented as gamma matrices $q^i = \gamma^i/\sqrt{2}$. The monopole Hilbert space is the representation space of the anti-commutation relation, and is hence a spinor representation of $SO(2n_f)$, with $2^{n_f}$ states. For the $SU(n_c)$ gauge theories ($n_c > 2$), however, the flavor symmetry is only as large as $U(n_f) \subset SO(2n_f)$, and hence the monopole multiplet (spinor under $SO(2n_f)$) is decomposed into irreducible multiplets under $U(n_f)$: totally anti-symmetric tensor representations. One can easily check that the the dimensions match: $\sum_r {}_{n_f}C_r = 2^{n_f}$ using the binomial theorem.

We have learned that the monopoles acquire flavor quantum numbers of the rank-$r$ totally anti-symmetric representations, while the theory breaks the $U(n_f)$ flavor symmetry dynamically to $U(r) \times U(n_f - r)$ in the previous section. We are naturally led to a conjecture that the flavor symmetry breaking is caused by the condensation of the magnetic monopoles which causes confinement at the same time.

## 4 Moduli Space of $N = 2$ Theories

The classical moduli space of the theory is determined by solving the vacuum equations

$$\Phi Q_i = 0, \tag{9}$$

$$\tilde{Q}_i \Phi = 0, \tag{10}$$

$$\sum_i \left\{ Q_i \tilde{Q}_i - \frac{1}{n_c} \mathrm{tr} Q_i \tilde{Q}_i \right\} = 0, \tag{11}$$

$$\left[\Phi, \Phi^\dagger\right] = 0, \tag{12}$$

$$Q^\dagger T^a Q - \tilde{Q}^T T^a \tilde{Q}^* = 0. \tag{13}$$

There are three types of "branches" to the vacuum solutions[6]: (1) Coulomb branch, (2) Non-baryonic (or mixed) branch, and (3) Baryonic branch. The baryonic branch appears only for $n_f \geq n_c$ and we will not discuss it.

The solution to Eq. (12) is given by $\Phi = \mathrm{diag}(\phi_1, \cdots, \phi_{n_c})$ with the constraint $\mathrm{tr}\Phi = \sum_k \phi_k = 0$. This defines the complex $(n_c - 1)$-dimensional Coulomb branch. At a generic point on the Coulomb branch, the theory is a free $U(1)^{n_c - 1}$ gauge theory while there appear massless particles on singular submanifolds. The singularities can be found where the auxiliary curve[12]

$$y^2 = \prod_{k=1}^{n_c} (x - \phi_k)^2 + 4\Lambda^{2n_c - n_f} \prod_{i=1}^{n_f} (x + m_i) \tag{14}$$

is maximally degenerate.

The non-baryonic branch is given by the following field configurations:

$$Q = \left(\begin{array}{ccc|ccc|c} \kappa_1 & & & 0 & & & 0 \\ & \ddots & & & \ddots & & \vdots \\ & & \kappa_r & & & 0 & 0 \\ \hline 0 & & & 0 & & & 0 \\ & \ddots & & & \ddots & & \vdots \\ & & 0 & & & 0 & 0 \end{array}\right) \tag{15}$$

$$\tilde{Q} = \left(\begin{array}{ccc|ccc|c} 0 & & & \kappa_1 & & & 0 \\ & \ddots & & & \ddots & & \vdots \\ & & 0 & & & \kappa_r & 0 \\ \hline 0 & & & 0 & & & 0 \\ & \ddots & & & \ddots & & \vdots \\ & & 0 & & & 0 & 0 \end{array}\right) \tag{16}$$

$$\Phi = \left(\begin{array}{ccc|ccc} 0 & & & & & \\ & \ddots & & & 0 & \\ & & 0 & & & \\ \hline & & & \phi_{r+1} & & \\ & 0 & & & \ddots & \\ & & & & & \phi_{n_c} \end{array}\right) \tag{17}$$

Because both hypermultiplets $Q$, $\tilde{Q}$ and the vector multiplet $\Phi$ have expectation values, it is also called the mixed branch. There are separate $r$-branches for each choice of the integer $r$. The integer $r$ can range from 1 to $\min\{[\frac{n_f}{2}], n_c - 2\}$, and hence this branch exists only for $n_f \geq 2$ and $n_c \geq 3$. It is important to note that the limit $\kappa_k \to 0$ recovers $U(r)$ gauge symmetry and the branch touches the Coulomb branch ("root" of the no-baronic branch). The theory at the root is a $U(r) \times U(1)^{n_c - r - 1}$ gauge theory which is asymptotically non-free, and hence the gauge fields survive as dynamical degrees of freedom in the low-energy limit. Along the root, there are special isolated points where we can find $n_c - r - 1$ massless monopole multiplets so that the curve becomes maximally degenerate.

## 5 Coulomb Branch Description

One can locate points on the Coulomb branch where the curve is maximally degenerate, so that the points survive after $\mu \neq 0$ perturbation. They do lie on the roots of $r$-branches. After mass perturbation for the hypermultiplets, one can count the number of vacua by identifing the points on the Coulomb brach which coalese to the same point when the masses are turned off. This is a technically involved analysis which required us many pages of the paper [2]. Nonetheless the result is simple. Starting from the maximally degenerate point on the $r$-branch root, the mass perturbation splits the point into ${}_{n_f}C_r$ vacua. Comparing this counting to the large $\mu$ analysis, we can say that the vacuum at the $r$-branch root breaks the flavor symmetry as $U(n_f) \to U(r) \times U(n_f - r)$.

Therefore, the following picture appears true. The semi-classical monopoles far away from the singularities on the Coulomb branch acquire the flavor quantum number of the rank-$r$ totally anti-symmetric tensor prepresentation under the $U(n_f)$ flavor group. They become massless at the maximally degenerate point along the $r$-branch root and condense upon $\mu \neq 0$ perturbation. This picture, however, leaves a paradoxical situation. The low-energy effective Lagrangian of the monopoles would have a large accidental symmetry $U({}_{n_f}C_r)$, and upon condensation of one of the components, all the others remain massless. Even though it is logically not impossible, it casts some doubts about this naive picture. Indeed, something non-trivial happens between the semi-classical regime and the $r$-branch root as I will describe in the next section.

## 6 Low-energy Effective Lagrangians

Now we aproach the singularities from the Higgs branch. As remarked earlier, by turning off $\kappa_k \to 0$, we can approach the roots where we recover infrared-free $U(r) \times U(1)^{n_c-r-1}$ gauge theory. At the maximally degenerate point along the roots, we have also $n_c - r - 1$ additional magnetic monopole hypermultiplets $e_k$, $\tilde{e}_k$ coupled to the $U(1)$ factors. The fundamental quarks still couple to the $SU(r)$ gauge factor as the fundamental representation $q_i$ and $\tilde{q}_i$ because the non-renormalization theorem guarantees the flat hyper-Kähler metric for the quarks. The unique effective Lagrangian obtained this way is[6]

$$W = \sqrt{2}\tilde{q}_i \phi q_i + \sqrt{2}\psi_0 \tilde{q}_i q_i + \sqrt{2} \sum_{k=1}^{n_c-r-1} \psi_k \tilde{e}_k e_k, \tag{18}$$

where $\phi$, $\psi_0$ belong to the $U(r)$ vector multiplet and $\psi_k$ to each of the $U(1)$ vector multiplets. The $N = 1$ perturbation then is given by

$$\Delta W = \mu\Lambda \sum_{j=0}^{n_c-r-1} x_j \psi_j + \mu \mathrm{tr}\phi^2, \tag{19}$$

where $x_j$ are $O(1)$ constants, and we find vacua

$$q = \tilde{q} = \left( \begin{array}{ccc|c} 1 & & & 0 \\ & \ddots & & \vdots \\ & & 1 & 0 \end{array} \right) \sqrt{-\frac{\mu\Lambda}{\sqrt{2}r}}, \tag{20}$$

$$e_k = \tilde{e}_k = \sqrt{-\mu\Lambda}, \quad \psi_0 = \psi_k = 0. \tag{21}$$

Note that the expectation values of $q$, $\tilde{q}$ break the flavor symmetry $U(n_f)$ to $U(r) \times U(n_f - r)$, where the unbroken $U(r)$ is the diagonal subgroup of the flavor group $U(n_f)$ and the $U(r)$ gauge group. And there are ${}_{n_f}C_r$ choices to pick $r$ flavors out of $n_f$ quark flavors for vacuum expectation values.

The semi-classical monopoles in the rank-$r$ anti-symmetric tensor representation therefore must have "broken up" into "dual quarks" in the way that the monopole $M_{i_1 \cdots i_r} = q_{i_1} \cdots q_{i_r}$ is matched to the baryonic composite before one reaches the singularities on the Coulomb branch. For the special case of $r = 1$, the "dual quarks" themselves are the magnetic monopoles. The evidence for this identification is the following. The singularities, after $m_i \neq 0$ perturbation, have $U(1)^{n_c-1}$ gauge group and one can study the monodromy around the singularities. It can be seen that there are one massless magnetic monopole for each $U(1)$ factors. By sending quark mass to zero, ${}_{n_f}C_r$ singularities coalesce into a point where the massless monopoles belong to the $U(r)$ quark multiplet. Therefore, the quarks, which are continuously connected to the "electric quarks" at the large VEVs along the Higgs branches, are indeed magnetic degrees of freedom at the non-baryonic branch roots.

## 7 Conclusion

We have studied the issues of confinement and the dynamical flavor symmetry breaking in gauge theories, by starting with $N = 2$ $SU(n_c)$ QCD with $n_f$ flavors and perturbing it by the adjoint mass term. We have shown that both confinement and flavor symmetry breaking are caused by a single mechanism: condensation of magnetic degrees of freedom which carry flavor quantum numbers.

We have also studied $USp(2n_c)$ theories. There the magnetic monopoles are spinors under the $SO(2n_f)$ flavor group, and cannot "break up" into quarks. Therefore quarks and monopoles coexist at the singularity on the moduli space and the theory becomes superconformal. No local effective Lagrangian can be written and one cannot discuss it along the same line as in the $SU(n_f)$ gauge theories. However, the flavor symmetry is broken to $U(n_f)$, and this is consistent with the condensation of the spinor monopoles. This strongly suggests that the overall picture of flavor symmetry breaking via monopole condensation is correct in this case as well.

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